Factor Structure of Mental Chronometric and Psychometric Assessment Instruments

A. Alexander Beaujean, Sean McGlaughlin, & Andy Knoop
Sponsoring Professor: Greg Holliday, Ph.D.
Assessment and Consultation Clinic
University of Missouri-Columbia

Data

Participants are 130 undergraduates with math difficulties who volunteered to participate in this research project. They were given four test batteries: (a) Wechsler Adult Intelligence Scale-III (WAIS-III), (b) the standard battery (minus S*tory Recall*) of the Woodcock-Johnson-III Tests of Achievement (WJ-III), (c) Wechsler Memory Scale-III (WMS-III), and (d) selected mental chronometric tasks (measured using the *Cognometer*). IQs ranged from 85 to 139, and ages ranged from 18-45 (70% were 24 years old or younger). The variables used were the 4-index scores of the WAIS-III (VCI, POI, WM, PS), 8 mental chronometric tasks, 6 WJ-III subtests, and 3 WMS-III index variables.

Analysis/Results

All factor analyses presented assume the factors obtained are orthogonal (i.e., not correlated).

Principle Axis Exploratory Factor Analysis

For the Principle Axis Exploratory Factor Analysis (PC EFA), Schmid and Leiman (1957) developed a procedure so data on different factor orders can be represented in a single matrix, showing the predicted loadings of variables on orthogonal factors at different orders. This procedure was used to produce Table 1, which shows factor loadings of the 21 variables on a 3rd-order factor, general cognitive ability (***g***), and four 2nd-order factors. All variables, except one, load substantially on ***g***, and all variables additionally only substantially load on one 2nd-order factor. All other loadings on the 2nd order factors approach zero except for a few, which, although not close to zero, are not as high as the values that mainly define the given factor. From this, it is clear that that the results support the standard multifactor view of cognitive abilities that postulates a general factor, ***g***, at the third stratum and a series of broad abilities at the second (Carroll, in press, 1993; Jensen, 1998). ***g*** explains the most common variance (33.37%). Of particular note, the mental chronometric (MC) tasks all load on ***g*** and one 2nd order factor (*Processing Speed*), which gives evidence that MC tasks measure ***g*** as well as many psychometric instruments, in addition to measuring a second latent variable, i.e., *processing speed*.

Maximum-Likelihood Analyses

The same data used for the PC EFA was submitted to a confirmatory factor analysis (CFA) with the AMOS 4 program. AMOS 4 is a structural equation modeling program that allows for maximum likelihood (ML) solutions of the factor structure. An advantage of using a ML algorithm over more traditional ones is that it allows for significance testing of the factor model.

A problem in submitting the EFA model in Table 1 to a ML solution is that Factor 5 (*Gsm/Glr*) only has two variables to define it (i.e., it is underidentified).To remedy this, the variable's loadings on the factor were constrained to be equal. When the CFA was run and tested, it was found that it did not fit the data extremely well, as evidenced by its high χ^2 (347.621, 171 *df*) and RMSEA (.089), and low IFI (.842) values. Consequently, a ML exploratory factor analysis (ML EFA) was performed to determine the model that would give the overall best fit. The results are shown in Figure 2. The difference in the CFA and ML EFA models are small, consisting only of constraining 3 more variables to be zero on the ***g*** factor, and freeing three variables on factors 3 (*Gq*) and 4 (*Gc*). These slight adjustments improved the model's fit considerably, as shown by the decreased χ^2 and RMSEA values, and increased IFI value.

Discussion

The results from submitting a group of psychometric and mental chronometric tasks to Principle Axis and a Maximum Likelihood Exploratory Factor Analyses conform highly with Carroll's (in press, 1993) three-stratum theory of cognitive abilities. Specifically, a general cognitive ability factor (***g***) can be extracted in which all the variables, except two simple mental chronometric tasks and two number fluency variables, substantially loaded. The reason the two number fluency variables did not load is most likely because the sample is made up of students who were self referred due to mathematics difficulties.

The major contributions of this study are twofold. First, all the MC tasks, except the aforementioned, loaded on ***g***. Because the sample consisted of an atypical sample, this gives confirmatory evidence of the factor invariance of mental chronometric tasks, namely that they measure both general cognitive ability as well as cognitive processing speed (Carroll, 1993; Jensen, 1993, 1998).

Second, in students with math difficulties, there is a specific group of tasks that measures number facility that consists of not only math-related items, but also working memory, processing speed, and mental chronometric tasks. This is pregnant with possibilities for the future of understanding and assessing people with mathematics difficulties, because it might allow for more direct and accurate measurement of the processing difficulties involved in mathematics disorders (Jensen, 1987).

References

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Table 1

Orthogonal Hierarchical Principle Axis Exploratory Factor Matrix for 21 Variables and 6 Factors

	Order:	2	1	1	1	1	
	Factor #:	*1*	*2*	*3*	*4*	*5*	**h^2**
	Factor Name:	*g*	*Gs*	*Gq*	*Gc*	*Gsm/Glr*	
	Variable:						
1	WAIS VCI	**0.505**	-0.094	-0.015	**0.580**	0.034	0.602
2	WAIS POI	**0.501**	0.139	**0.386**	0.064	0.046	0.425
3	WAIS WM	**0.526**	0.015	**0.434**	0.122	0.049	0.482
4	WAIS PS	**0.372**	**0.531**	0.163	-0.116	0.057	0.464
5	WMS General Memory	**0.355**	-0.080	0.019	0.028	**0.788**	0.754
6	WMS Working Memory	**0.469**	0.144	**0.449**	-0.090	0.174	0.481
7	WMS Immediate Memory	**0.364**	-0.060	0.001	0.012	**0.852**	0.862
8	WJ-III Letter-Word Identification	**0.421**	-0.001	-0.051	**0.429**	0.115	0.377
9	WJ-III Passage Comprehension	**0.610**	0.050	0.030	**0.620**	-0.099	0.770
10	WJ-III Reading Fluency	**0.397**	**0.285**	-0.047	0.201	0.163	0.308
11	WJ-III Calculation	**0.328**	-0.102	**0.562**	-0.112	0.017	0.447
12	WJ-III Applied Problems	**0.504**	-0.167	**0.620**	0.150	-0.177	0.720
13	WJ-III Math Fluency	**0.235**	0.172	**0.261**	-0.087	-0.024	0.161
14	Cog Physical Reflexes (SRT)	**0.181**	**0.397**	-0.060	-0.029	0.053	0.198
15	Cog Perceptual Reflexes (IT 1)	**0.284**	**0.561**	-0.061	0.039	-0.077	0.406
16	Cog Running Memory Capacity/Speed	**0.313**	**0.673**	-0.021	-0.035	-0.080	0.559
17	Cog Cognitive Reflexes	**0.331**	**0.593**	-0.001	0.032	-0.115	0.476
18	Cog Working Memory Speed	**0.443**	**0.543**	-0.041	0.113	0.100	0.516
19	Cog Perceptual Threshold/Acuity	**0.210**	**0.462**	0.006	-0.023	-0.105	0.269
20	Cog Delayed Working Memory Capacity	0.142	**0.340**	0.007	-0.046	-0.048	0.140
21	Cog Ln(Visual-Spatial)	**0.404**	**0.448**	0.078	0.038	0.072	0.377
	SMSQ	3.268	2.594	1.355	1.051	1.523	9.792
	%CCV	33.37%	26.49%	13.84%	10.73%	15.55%	100.00%

Note. Variable names are on left. Salient loadings of variables on common factors are in **bold**. The Factor number is *italicized*. The order of the factor is underlined. h^2: Communality of the variable. SMSQ: Sums of Squares. %CCV: Percentage of Common Variance. Factor Names: Gs: Processing Speed; Gq: Mathematics; Gc: Comprehension-Knowledge; Gsm/Glr: Short-Term/Long-Term Memory Composite

Table 2

AMOS Estimates of Orthogonal Hierarchical Factor Loadings for "Best Fit" Model for 21 Variables and 6 Factors

	Order:	2	1	1	1	1	
	Factor #:	*1*	*2*	*3*	*4*	*5*	**h^2**
	Factor Name:	*g*	*Gs*	*Gq*	*Gc*	*Gsm/Glr*	
	Variable:						
1	WAIS VCI	0.470	---		0.630	---	0.618
2	WAIS POI	0.566	---	0.339	---	---	0.435
3	WAIS WM	0.645	---	0.391	---	---	0.569
4	WAIS PS	0.345	0.550	---	---	---	0.422
5	WMS General Memory	0.381	---	---	---	0.860	0.885
6	WMS Working Memory	0.763	---	0.371	-0.423	---	0.899
7	WMS Immediate Memory	0.397	---	---		0.789	0.780
8	WJ-III Letter-Word Identification	0.397	---	---	0.375	---	0.298
9	WJ-III Passage Comprehension	0.577	---	---	0.632	---	0.732
10	WJ-III Reading Fluency	0.386	0.314	---	---	---	0.248
11	WJ-III Calculation	---	---	0.881	---	---	0.776
12	WJ-III Applied Problems	0.313	---	0.734	0.342	---	0.754
13	WJ-III Math Fluency	---	0.333	0.393	---	---	0.265
14	Cog Physical Reflexes (SRT)	---	-0.508	---	---	---	0.258
15	Cog Perceptual Reflexes (IT 1)	-0.185	-0.619	---	---	---	0.417
16	Cog Running Memory Capacity/Speed	-0.285	-0.671	---	---	---	0.531
17	Cog Cognitive Reflexes	-0.363	-0.557	---	---	---	0.442
18	Cog Working Memory Speed	-0.499	-0.496	---	---	---	0.495
19	Cog Perceptual Threshold/Acuity	-0.135	-0.529	0.233	---	---	0.352
20	Cog Delayed Working Memory Capacity	---	-0.448	---	---	---	0.201
21	Cog Ln(Visual-Spatial)	-0.523	-0.385	---	---	---	0.422
	SMSQ	3.487	2.788	1.929	1.233	1.362	10.799
	%CCV	32.29%	25.82%	17.86%	11.42%	12.61%	100.00%

Note. Measures of Goodness of Fit for the Whole Model:
χ^2 (69 *df*) = 277.029 ($p < .000$); Incremental Fit Index = 0.904;
RMSEA = 0.070, 90% CI: (.055, .085).